Two Dimensional Sensor Localization Using mN/2 Algorithm in Different Types of Distributed Fields

Serap Karagol and Dogan Yildiz

***Abstract*— Wireless Sensor Network (WSN) refers to a group of locationally dispensed and dedicated sensors that observe and record physical and environmental conditions and coordinate the aggregated data at a centrical location. To serve new applications, localization is largely used in WSNs to define the current location of the sensor nodes. In this paper, first, the proposed mN/2 algorithms performance compared with GPS, 3N, 3/2N and 3/2N(2) algorithms. The mN/2 algorithm is especially effective in very sparse networks where other algorithms usually fail. Even when the algorithm cannot locate a given node, it produces a polygonal estimate of the region in which the node is located. Monte Carlo simulations show that this algorithm performs better than other algorithms. Secondly, Uniform, Beta, Weibull, Gamma and Generalized Pareto distributed networks were used for localization using the mN/2 algorithm. The localization performance of the networks are evaluated and compared using MATLAB simulations.**

***Keywords*— Graph-Theory, Localization, Probability Distributions, Wireless Sensor Networks.**

I. Introduction

Technological developments in wireless systems allow for the emergence of sensor nodes that are cheap, capable of fulfilling multiple functions, and have low power consumption. A wireless network structure can be built by deploying a certain number of sensor nodes to a certain area. A typical Wireless Sensor Network (WSN) has basically two functions, including collecting information from each sensor node and processing this information according to the purpose of its intended use [1]. WSNs have extensive application areas. A WSN is used by rescue teams to locate and prioritize rescue requirements of avalanche victims [2], to monitor the vital signs of patients in hospital environments [3], to observe temperature where temperature change is important [4], in military applications [5], and many other areas.

For all of WSN's applications, the network needs to have the ability to determine the location of these sensor nodes for the purpose of managing the work of components properly. This situation reveals the localization problem within the sensor network. In the literature, various approaches have been put forward to solve this problem. Pandey et al. [6] constructed a localization table according to the energy of sensor nodes. Estimates on the distance between adjacent nodes are performed according to the energy available in the nodes. Kai et al. [7] presented a scalable range-free algorithm for localization in WSN. Their algorithm finds neighbor nodes by comparing Received Signal Strength (RSS) and estimates the location from these neighbors' location.

Some methods were developed based on graph theory for solving the localization problem. The use of graph-theoretic based notions in the localization of a sensor network is well described, and their importance from both the algorithmic and the analytic aspects is well demonstrated in recent research [8]-[9]. A feasible node localization scheme is presented by Liu et al. [9] incorporating the graph rigidity concept, and a new idea of B-N tree is introduced in the construction of a localizable collaborative body. Nussbaum et al. [10] considered the question of finding proper distance-preserving subgraphs, and the problem of partitioning a simple graph into an arbitrary number of distance-preserving subgraphs for clustering purposes. They also present a clustering algorithm called DP-Cluster, based on the notion of distance-preserving subgraphs. Vahidnia et al. [11] employed a new graph-theory and improved genetic algorithm based practical method to solve the optimal sectionalizer switch placement problem. A paradigm for a sensor network that tracks moving objects based on graph-theoretic sensor signal sequences in the time domain was proposed by Zheng et al. [12]. It proved that a triangular grid can track an object with error limited to a small neighborhood. An enhancing node-robustness algorithm using node diversity is proposed in [13]. According to node vulnerabilities, this algorithm can mitigate the exploitation of these vulnerabilities and the propagation of concerning attacks. Nakayama et al. [14] introduced tie-set graph theory and its application to smart grid networks. In the literature, statistical analysis related to both localization and the energy problem in wireless sensor networks are available in many studies. Kamyabpour et al. [15] use statistical tools to analyze dependency between Wireless Sensor Network (WSN) parameters and overall energy consumption. In this study, three statistical approaches (linear and non-linear correlation, p-value) are implemented to the consequence of detecting phase to extract the most efficacious parameters on WSN comprehensive energy consumption. The distribution of range estimation error is analyzed by Rasool et. al [16] using both graphical and computational goodness of-fit

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techniques, including empirical cumulative distribution function plotting, quantile–quantile plotting, probability density function plotting, kurtosis (K) test, skewness (S) test, linear correlation coefficient (γ) test, Anderson–Darling (A^2) test and chi-squared ($\chi 2$) test. They proposed the range infiltration algorithm (RFA), which is based on the A^2 test and filters out the range estimations with high errors. In [17], equipped with moments, the optimal fusion rule (OFR) distribution is approximated by Gaussian and Gamma distributions via moment mapping method. They showed that the Gamma distribution fits the OFR distribution to a high extent when compared with the Gaussian distribution. Tae Hong et al. [18] propose a new data filtering schema based on statistical data analysis. Through performance analysis, they show that the proposed schema does better than the Kalman filtering schema in terms of the number of messages transmission. In [19], the authors present the SA-TC algorithm for detecting and defending against this serious threat. It is based on the on-demand multi-path routings and uses statistical analysis and time constraint to identify the suspected links. Tsai et al. [20] report different aspects of a statistical analysis of four representative in-car wireless channels based on the received power data collected from a Binary Phase Shift Keying (BPSK) transmission experiment. They used Rayleigh, Log normal, Nakagami, Rice, and Weibull distributions in their study.

Lastly, a uniformly distributed network was used to localize the target nodes while mN/2 algorithm was being run. In this paper, in addition to the use of the Uniform distributed field for observing mN/2 algorithms performance, Beta, Weibull, Gamma and Generalized Pareto distributed networks were also used for localization. The localization performance of the networks were evaluated and compared using MATLAB simulations.

II. Proposed mN/2 Algorithm

A. Chains

In this section, a new concept of specific multi-hop paths called chains is introduced. They are helpful for the construction of polygons which are used in the calculation of the final resultant polygonal area. Dijkstra's algorithm [8] finds the shortest paths from the target node to every other node in the graph containing target nodes. If two such anchor nodes A_1 and A_2 exist and an optimal multi hop shortest path that connects the first anchor A_1 through the target node to the second anchor A_2, then A_1 –S (target node) - A_2 is called a chain, as illustrated in Fig. 1. It is assumed that there are multiple hops from A_1 - S (target node) and also multiple hops from S (target node) - A_2. Two circles are constructed at the center of the anchor locations A_1 and A_2 and with radii R_1 and R_2 respectively. R_1 and R_2 are the multi-hop distances from the target node to the anchor nodes A_1 and A_2 respectively.

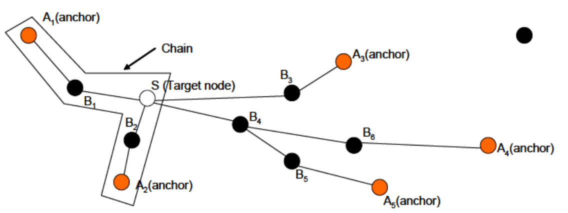

Fig. 1. Chains

In Fig. 1, there are 5 anchor nodes $A_1,\ldots,A_5$ the target node S; and intermediate nodes $B_1,\ldots,B_6$. The shortest paths from S to all nodes are shown in Fig.1. Based on this node placement, there are 10 chains can be constructed as shown in Table 1.

TABLE I
CONSTRUCTED CHAINS FOR NODES SHOWN IN FIG.1

Chain Number	Circle (Center, Radius)	
1	Chain	$A_1-B_1-S-B_2-A_2$
	1st Circle	$(A_1, \|\|A_1-B_1\|\|+\|\|B_1-S\|\|)$
	2nd Circle	$(A_2, \|\|A_2-B_2\|\|+\|\|B_2-S\|\|)$
2	Chain	$A_1-B_1-S-B_3-A_3$
	1st Circle	$(A_1, \|\|A_1-B_1\|\|+\|\|B_1-S\|\|)$
	2nd Circle	$(A_3, \|\|A_3-B_3\|\|+\|\|B_3-S\|\|)$
3	Chain	$A_1-B_1-S-B_4-B_6-A_4$
	1st Circle	$(A_1, \|\|A_1-B_1\|\|+\|\|B_1-S\|\|)$
	2nd Circle	$(A_4, \|\|A_4-B_6\|\|+\|\|B_6-B_4\|\|+\|\|B_4-S\|\|)$
4	Chain	$A_1-B_1-S-B_4-B_5-A_4$
	1st Circle	$(A_1, \|\|A_1-B_1\|\|+\|\|B_1-S\|\|)$
	2nd Circle	$(A_5, \|\|A_5-B_5\|\|+\|\|B_5-B_4\|\|+\|\|B_4-S\|\|)$
5	Chain	$A_2-B_2-S-B_3-A_3$
	1st Circle	$(A_2, \|\|A_2-B_2\|\|+\|\|B_2-S\|\|)$
	2nd Circle	$(A_3, \|\|A_3-B_3\|\|+\|\|B_3-S\|\|)$
6	Chain	$A_2-B_2-S-B_4-B_6-A_4$
	1st Circle	$(A_2, \|\|A_2-B_2\|\|+\|\|B_2-S\|\|)$
	2nd Circle	$(A_4, \|\|A_4-B_6\|\|+\|\|B_6-B_4\|\|+\|\|B_4-S\|\|)$
7	Chain	$A_2-B_2-S-B_4-B_5-A_5$
	1st Circle	$(A_2, \|\|A_2-B_2\|\|+\|\|B_2-S\|\|)$
	2nd Circle	$(A_5, \|\|A_5-B_5\|\|+\|\|B_5-B_4\|\|+\|\|B_4-S\|\|)$
8	Chain	$A_3-B_3-S-B_4-B_6-A_4$
	1st Circle	$(A_3, \|\|A_3-B_3\|\|+\|\|B_3-S\|\|)$
	2nd Circle	$(A_4, \|\|A_4-B_6\|\|+\|\|B_6-B_4\|\|+\|\|B_4-S\|\|)$
9	Chain	$A_2-B_2-S-B_4-B_5-A_5$
	1st Circle	$(A_3, \|\|A_3-B_3\|\|+\|\|B_3-S\|\|)$
	2nd Circle	$(A_5, \|\|A_5-B_5\|\|+\|\|B_5-B_4\|\|+\|\|B_4-S\|\|)$
10	Chain	$A_4-B_4-B_6-S-B_4-B_5-A_5$
	1st Circle	$(A_4, \|\|A_4-B_6\|\|+\|\|B_6-B_4\|\|+\|\|B_4-S\|\|)$
	2nd Circle	$(A_5, \|\|A_5-B_5\|\|+\|\|B_5-B_4\|\|+\|\|B_4-S\|\|)$

If there are N_A anchor nodes, then the following number of chains can be calculated.

$$N_H = \sum_{j}^{N_A-1} j = \frac{1}{2} N_A (N_A - 1) \cdot \quad (1)$$

If 100 anchor nodes exist that are connected, one must check about (100×99)/2=4950 chains. Note that the intersection can be computed recursively at a great reduction in computational complexity.

Let C_j^1 and C_j^2 be the two circles defined by Chain j. Note that if the two circles C_{j1}^x and C_{j2}^x (where x means Do Not Care) are centered on the same anchor A_k then they are the same circle because the radius is simply the sum of the distance of the nodes along the shortest path that connects S to some anchor A_k where k=1,2,…,n. Then the target will be ultimately located with the region

$$U = \bigcap_{j=1}^{N_H} (C_j^{\ 1} \cap C_j^{\ 2}) \\ = (C_1^{\ 1} \cap C_1^{\ 2}) \cap (C_2^{\ 1} \cap C_2^{\ 2}) \ldots, \\ = \bigcap_{j=1}^{N_A} Circle(A_j, d_j) \tag{2}$$

where

$$d_j = \sum_{k=1}^{L_j} \| b_k - b_{k-1} \| . \tag{3}$$

The optimal path $b_0^j - b_1^j - \ldots - b_k^j$ denotes the shortest path from the anchor $A_j = b_0^j$ to the target $S = b_{Li}^j$ where there are L_j intermediate nodes $b_1, \ldots b_{Lj-1}$.

In short, if there are N_A anchor nodes that are connected in a graph-theoretic multihop manner to the target S; then S can be localized in the intersection of N_A circles.

Computationally, the algorithm proceeds by approximating each circle using a regular polygon P_j :

1. Initilialize $U^{(1)}$=Polygon(A_1,d_1)
2. For j=2:N_A
   a) $U^{(j)} = U^{(j-1)} \cap$ Polygon(A_j,d_j)

B. Polygons and Their Intersections

The intersection of these circles (A_i, d_i) where A_i is the absolute location of the anchor node and the d_i is the multi-hop distance that forms the radius of the circles that can be approximated as a polygon in which the target node must be present (target node, A_{i1}, A_{i2}, …, A_{in}). Here, n is the number of all anchor nodes to which the target node is connected. In the node detection step, the proposed algorithm computes the intersection of polygons, and involves the final decision making where one/ both the candidate locations are inside/outside the polygonal area as shown in Fig. 2.

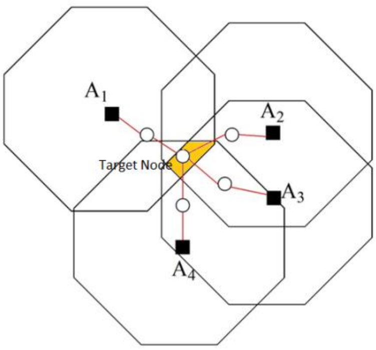

Fig. 2. The resultant polygonal area of the node location formed by the intersection of the polygons.

If one of the candidate locations are found to be inside the resultant polygonal area, then that candidate is deduced to be the actual location of the node. If both candidates are found to be inside the resultant polygonal area, the ambiguity cannot be cleared.

All the polygons are convex. This significantly simplifies the computation of the intersections, since the boundaries of two convex polygons can not intersect non-trivially more than twice. A simple algorithm is to start at one vertex of the first polygon which is inside (or outside) the second polygon and march through adjacent vertices of the first polygon until the boundary of the second polygons is crossed twice. If the marching comes back to the initial node without crossing the boundary, then the two polygons do not intersect.

mN/2 algorithm employs a multi-hop, cooperative, graph-theoretic based localization approach to localize many nodes that would otherwise remain unlocalized. It first follows the procedure similar to the one used by Barbeau et al. [21], of using two anchor nodes to find the two candidate locations, and then moves on to resolve the ambiguity and to eliminate the false candidate location using the other anchor nodes' locations and their multi-hop distances. The neighboring nodes that are employed in mN/2 algorithm contain a mixture of in-range but unlocalized nodes, and the anchor nodes that are multiple hops away. The algorithm consists of following steps:

1. Compute the shortest paths from each of the target nodes to a priori known anchor nodes locations with well known Dijkstra algorithm [8].
2. Generate circles centered at all anchor nodes that are K- hops away, and whose radii are bounded by a KR, where R is the shortest distance from the target node under consideration to the anchor.
3. Intersect the generated polygons to compute the resultant polygon.
4. Clear the ambiguity of which candidate location is the actual target node depending on whether one candidate/both candidates are inside/outside the resultant polygonal area.

III. Fields in Different Distributions

A. Uniform Distribution

One of the easiest continuous distributions in all of statistics science is the continuous uniform distribution, and this distribution was used for different applications. In [22], a method for measuring the productivity level of a decision making unit when it is in a negative situation, as well as estimating the productivity using uniform distribution, is shown. In [23], for a directed graph whose underlying undirected graph is tidy, the authors demonstrated that whether the uniform distribution on the vertices of the graph is an immobile distribution depends on a local characteristic of the graph, namely if (u, v) is a directed edge, then out-degree (u) is equal to in-degree (v).

This distribution is characterized by a density function that is "flat" and thus the probability is uniform in a closed interval, say [A,B]. The density function of the continuous uniform random variable X on the interval [A, B] is

$$f(x) = \begin{cases} \dfrac{1}{B-A} & , \quad A \le x \le B \\ 0 & , \quad \text{elsewhere} \end{cases} . \tag{4}$$

The density function composes a rectangle with base B−A and height 1/B-A. As a result, the uniform distribution is generally called the rectangular distribution [24]-[25]. Note, however, that the interval may not always be closed: [A, B].

It can be (A, B) as well. The density function for a uniform random variable on the interval [1, 3] is shown in Fig. 3.

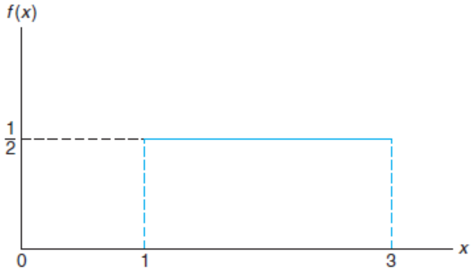

Fig. 3. The density function for a random variable on the interval [1, 3] [24].

The mean and variance of the uniform distribution [25] are

$$\mu = \frac{A+B}{2} \quad \text{and} \quad \sigma^2 = \frac{(B-A)^2}{12}. \tag{5}$$

Fig. 4. shows Uniform distribution of 100 nodes. Blue circle nodes and red square nodes represent position-aware and non-position-aware nodes, respectively.

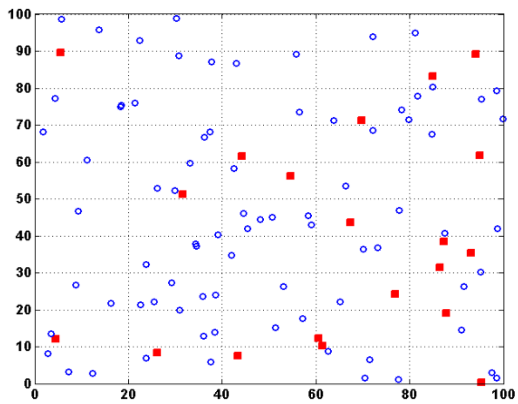

Fig. 4. Uniform distribution of 100 nodes

B. Beta Distribution

A beta function is defined by

$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx \qquad , \tag{6}$$
$$= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \qquad \text{for } \alpha,\beta > 0$$

where $\Gamma(\alpha)$ is the gamma function.

The continuous random variable X has a beta distribution with parameters α >0 and β>0 if its density function is given by

$$f(x) = \begin{cases} \dfrac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} & , \quad 0 < x < 1 \\ 0 & , \quad \text{elsewhere} \end{cases}. \tag{7}$$

Note that the uniform distribution on (0,1) is a beta distribution with parameters $\alpha = 1$ and $\beta = 1$.

The mean and variance of a beta distribution with parameters α and β are

$$\mu = \frac{\alpha}{\alpha+\beta} \quad \text{and} \quad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}, \tag{8}$$

respectively [4].

The beta distribution is a probability distribution described in an interval [0 1], parameterized by two shape parameters α and β. The beta distribution has an advance over other probability distributions in that its domain is bounded and it procures various shapes depending on its parameters: flat, convex, concave and slanted. When α = β, the distribution is symmetric about x = ½ [26].

Fig. 5. shows Beta distribution of 100 nodes. Two parameters of Beta function, α and β, are choosen as 4 and 2 respectively. Asymmetric distributions are obtained by choosing alpha and beta to be different.

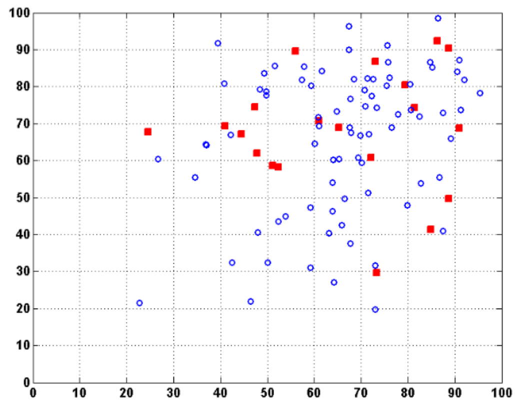

Fig. 5. Beta distribution of 100 nodes

C. Weibull distribution

Modern technology has enabled engineers to design many sophisticated systems whose process and safety depend on the reliability of the several components making up the systems. For example, a steel column may buckle, a fuse may burn out, or a heat-sensing device may fail. Alike components subjected to alike environmental situations will fail at different and imponderable times [25]. Weibull Statistical Distribution is also a prevalent method for examining wind speed measurements and specifying wind energy potential. Weibull probability density function can be used to predict wind density, wind energy potential, and wind speed [27]-[29].

The continuous random variable X has a Weibull distribution, with parameters α and β, if its density function is given by

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}} & , \quad x > 0 \\ 0 & , \quad \text{elsewhere} \end{cases}, \tag{9}$$

where α, β>0.

The graphs of the Weibull distribution for α = 1 and various values of the parameter β are illustrated in Fig. 6. It can be seen from the figure that the curves change highly in shape for different values of the parameter β. If β = 1 taken, the Weibull distribution changes to the exponential distribution. For values of β > 1, the curves become somewhat bell shaped and look like the normal curve but display some curvature.

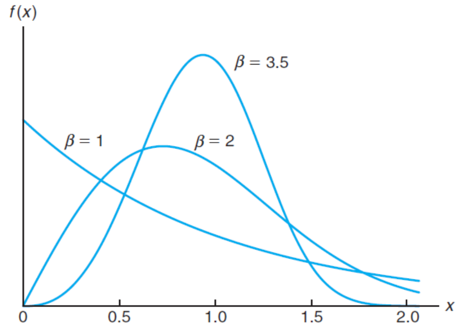

Fig. 6. Weibull distributions (α = 1) [24].

The mean and variance of the Weibull distribution are

$$\mu = \alpha^{-1/\beta}\Gamma(1+\frac{1}{\beta})$$
$$\sigma^2 = \alpha^{-2/\beta}\{\Gamma(1+\frac{2}{\beta}) - [\Gamma(1+\frac{1}{\beta})]^2\} \quad . \quad (10)$$

Fig. 7 shows Weibull distribution of 100 nodes. This distribution has two parameters which k > 0 is the shape parameter and λ > 0 is the scale parameter of the distribution. k and λ are chosen as 1 and 0.12 respectively for this simulation.

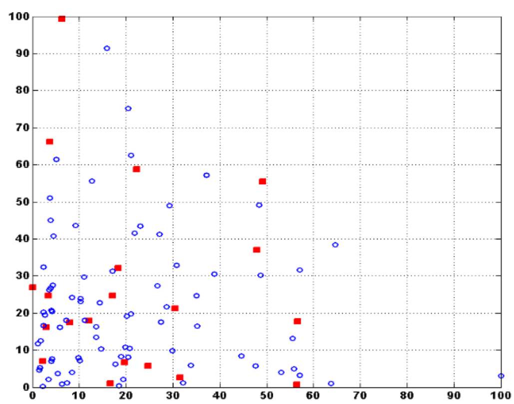

Fig. 7. Weibull distribution of 100 nodes

D. Gamma Distribution

The gamma distribution derives its name from the well-known gamma function, studied in many areas of mathematics. The gamma function is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx, \quad \text{for } \alpha > 0 \cdot \quad (11)$$

The continuous random variable X has a gamma distribution, with parameters α and β, if its density function is given by

$$f(x) = \begin{cases} \frac{1}{\beta\alpha\Gamma(\alpha)} x^{\alpha-1}e^{-x/\beta} & , \quad x > 0 \\ 0 & , \quad \text{elsewhere} \end{cases} , \quad (12)$$

where α>0 and β>0 [25]-[30].

Graphs of several gamma distributions are shown in Fig. 8. for certain determined values of the parameters α and β. The special gamma distribution for which α = 1 is called the exponential distribution [25].

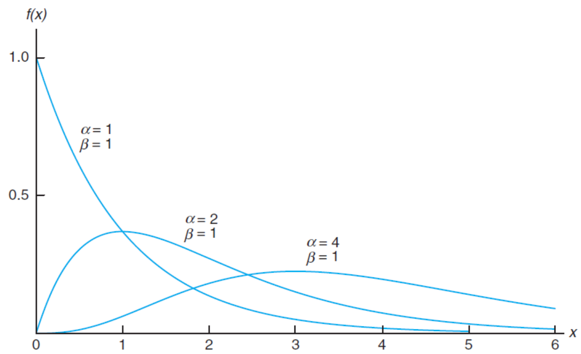

Fig. 8. Gamma distributions [24]

Fig. 9 shows Gamma distribution of 100 nodes.

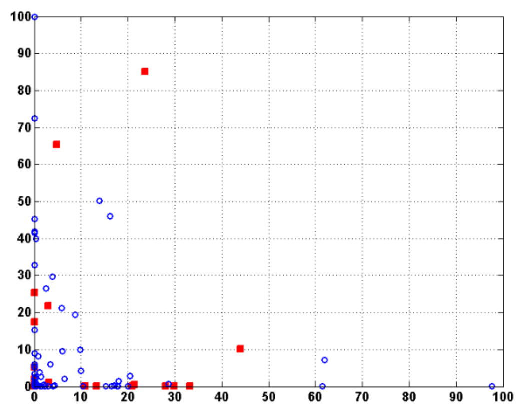

Fig. 9. Gamma distribution of 100 nodes

E. Generalized Pareto distribution

The Generalized Pareto distribution introduced by

$$F(q) = 1 - e^{-(q-q_0)/\alpha}, \quad \kappa = 0 \quad (13)$$

$$F(q) = 1 - (1 - \kappa\frac{q-q_0}{\alpha})^{1/\kappa} \quad \kappa \neq 0, \quad (14)$$

where α is the scale parameter, κ is the shape parameter, and q_0 is the threshold [31].

Fig. 10. shows Pareto distribution of 100 nodes. Three parameters of Pareto function, tail index (shape, κ), scale parameter α and threshold (location) parameter q_0, are chosen as 0.1, 0.1 and 1 respectively. When $\kappa > 0$ and theta is equal to α / κ, the Generalized Pareto is equivalent to the Pareto distribution.

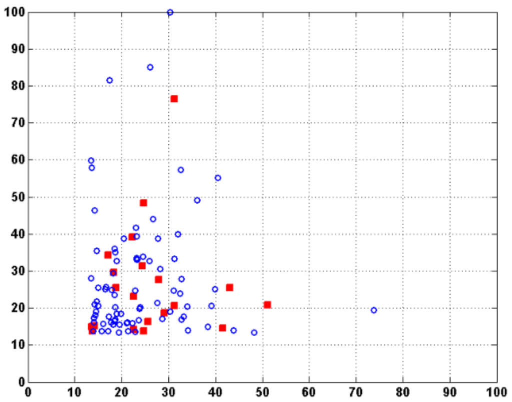

Fig. 10. Pareto distribution of 100 nodes

IV. SIMULATION RESULTS

The proposed graph-theoretic algorithm is found to be extremely effective in anchor-sparse environments. In an anchor-sparse environment, the availability of anchor nodes is minimal, even if the actual target nodes density is significant.

In the example presented, a particular field topology is set up in simulation. Extensive simulations are conducted in an environment similar to the one created by Barbeau and Kranakis [21]. The sensors are spread in a unit square independently with uniform distribution. The reachability range of each node is as given by the Eq. (15).

$$r = \sqrt{\frac{\log n + k \log \log n + \log(k!) + c}{n\pi}}. \quad (15)$$

The constants k and c are given a value of 1 and n is the number of deployed nodes in the network. The square field of 100x100 units, a communications range of 15% units of the side dimension of the field, and an average anchor density of 9% of total nodes is assumed. Simulations show that 3 Neighbor algorithm and 3/2 Neighbor algorithm are unable to find any target nodes as shown in Fig. 11.

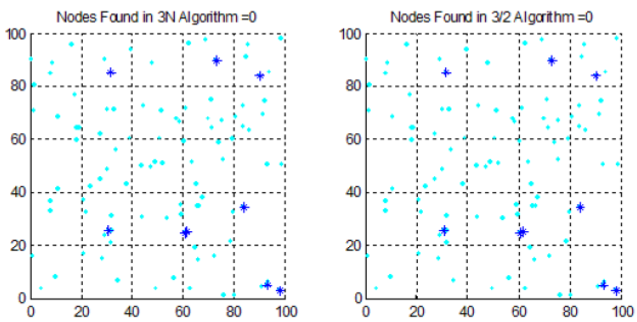

Fig. 11. 3N and 3/2 N algorithms fail to find any nodes

When the proposed algorithm is exclusively run on this field signature, it helps localize eight target nodes due to its inherent capability. Using other target nodes as intermediate nodes utilizes its cooperative multi-hop localization technique, shown here as green nodes in Fig. 12.

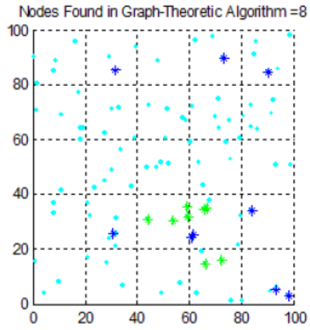

Fig. 12. The proposed algorithm succeeds in finding eight nodes.

The Monte-Carlo simulation is run 200 times with different field signatures and the results are averaged for each network size. The proposed mN/2 algorithm outperforms other algorithms by almost 200% which is expected due to the inherent capability to use the target anchor nodes in the chains mentioned before in the multi-hop cooperative localization of the nodes. It provides the position estimates in an extremely sparse environment of only containing 9% of the anchor nodes on average. Fig. 13 shows the results of the simulation of 10 to 50 node networks run 200 times for each network size.

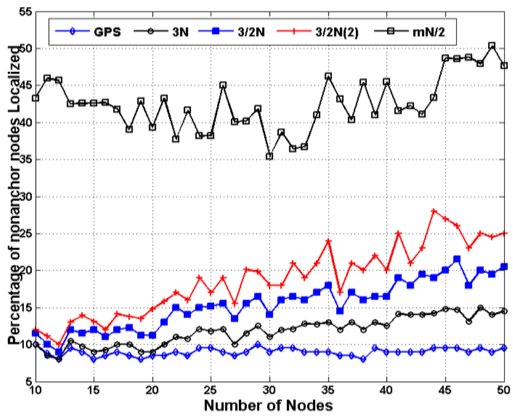

Fig. 13. An average of 9% anchor nodes present in the field

In anchor-dense environments, the availability of the anchor nodes is relatively high, as in the scenario of 28% anchor nodes of the total nodes shown in Fig. 14. The mN/2 algorithm showed an improvement of 25%-30% more nodes localized, compared to other algorithms.

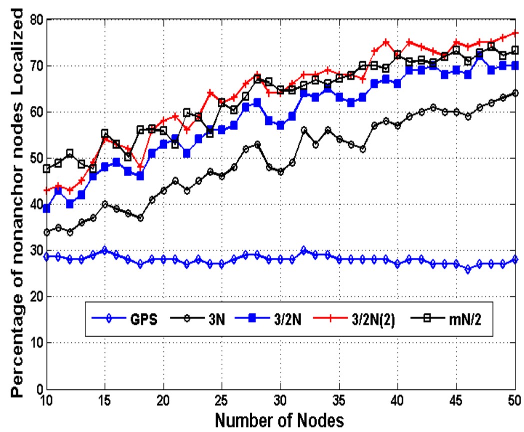

Fig. 14. An average of 28% anchors nodes present in the field

Uniformly distributed network was used to localize the target nodes to observe the mN/2 algorithms performance. But in this paper, Uniform, Beta, Weibull, Gamma and Generalized Pareto distributed networks were used for localization, and the localization performance of the networks are evaluated and compared using MATLAB simulations. Fig. 15 is produced by varying the number of total nodes from 10 to 50 for a dynamic communication range depending on the number of total nodes for uniform distribution. X-axis is the number of nodes and y-axis is the percentage of target nodes localized. mN/2 algorithm is run on Beta, Weibull, Gamma and Pareto distributed environments, as shown in Fig. 16, Fig. 17, Fig. 18 and Fig. 19 respectively. Generally, for all of the distributions, mN/2 algorithm has better results in the 28% anchor percentage distributed fields than in the 9% anchor percentage distributed fields. And with the increasing number of nodes, the localization performance of mN/2 algorithm increases for all of the distributions.

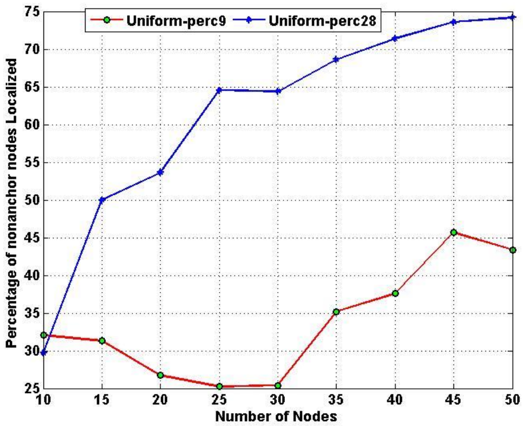

Fig. 15. Percentage of target nodes localization for uniform distribution

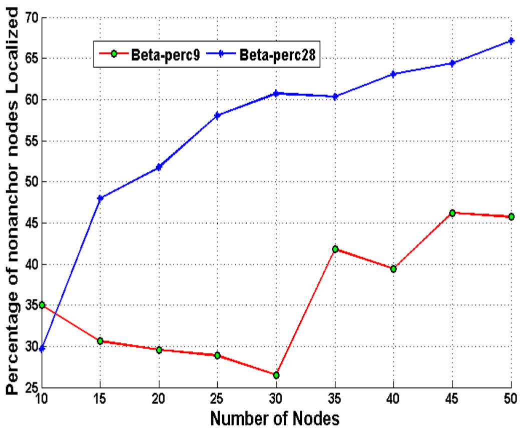

Fig. 16. Percentage of target nodes localization for beta distribution

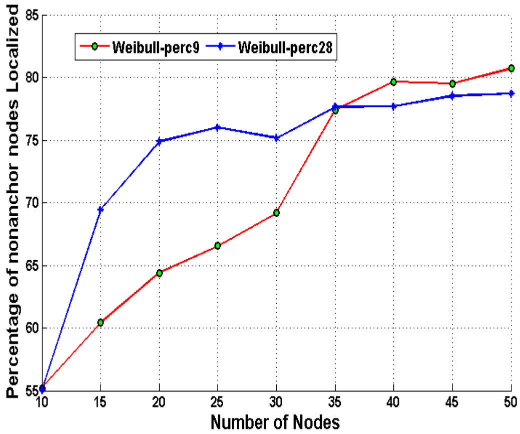

Fig. 17. Percentage of target nodes localization for weibull distribution

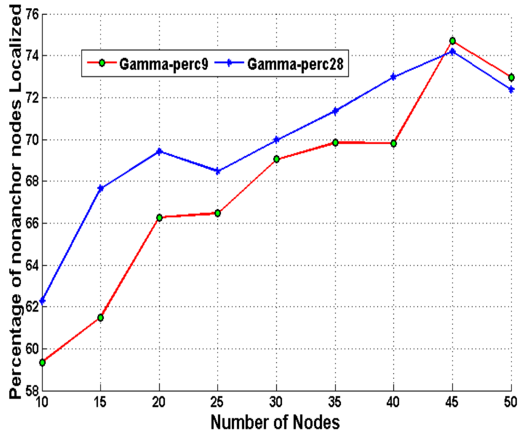

Fig. 18. Percentage of target nodes localization for gamma distribution

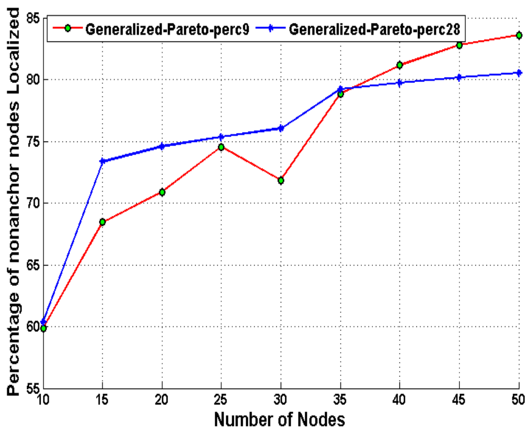

Fig. 19. Percentage of target nodes localization for pareto distribution

Fig. 20 and Fig. 21 show the comparison of 5 distributed environments with different number of nodes. Among all distributions, localization of Pareto distributed nodes shows the best result for all simulations.

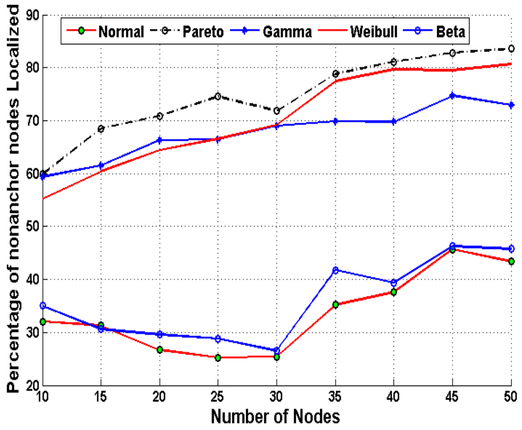

Fig. 20. Comparison of distributions with varying number of total nodes for 9% anchor nodes

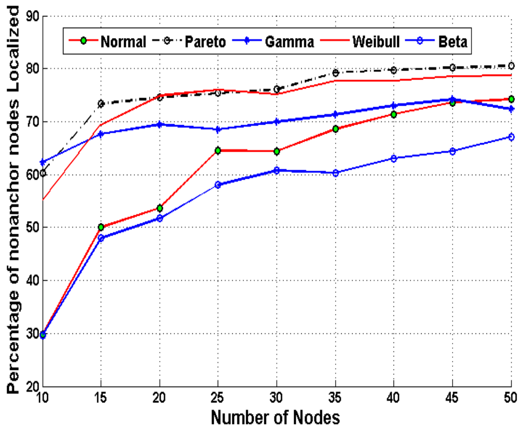

Fig. 21. Comparison of distributions with varying number of total nodes for 28% anchor nodes

V. CONCLUSION

A novel graph-theoretic, cooperative, multi-hop localization algorithm is proposed in this work. It can resolve several ambiguities as to the localization of nodes, and is vastly superior when in sparse networks. Detailed Monte-Carlo simulation on the same sensor field distributions have shown an excellent performance of the proposed anchor-sparse environments, where other well known algorithms fail to produce meaningful results. In very low anchor-density networks, the proposed algorithm showed close to 200 percent localization improvement over 3N and 3/2N algorithms. Moreover, the proposed algorithm can produce polygonal estimates of the locations of all nodes, even if those nodes are not locatable theoretically. And secondly, the mN/2 algorithm is tested on an environment created with Uniform, Beta, Weibull, Gamma, and Pareto distributions. Generally, for all the distributions, mN/2 algorithm has better results in the 28% anchor percentage distributed fields than in the 9% anchor percentage distributed fields. And with the increasing number of nodes, the localization performance of mN/2 algorithm increases for all the distributions. Among all distributions, localization of Pareto distributed nodes shows the best result for all simulations. The sensors that detect the movements of the objects are not considered in this paper. They will be addressed in future work.

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